



ENGINEERING MECHANICS I:

ANALYSIS OF SIMPLE STRUCTURES

Compiled by: Solomon Getachew



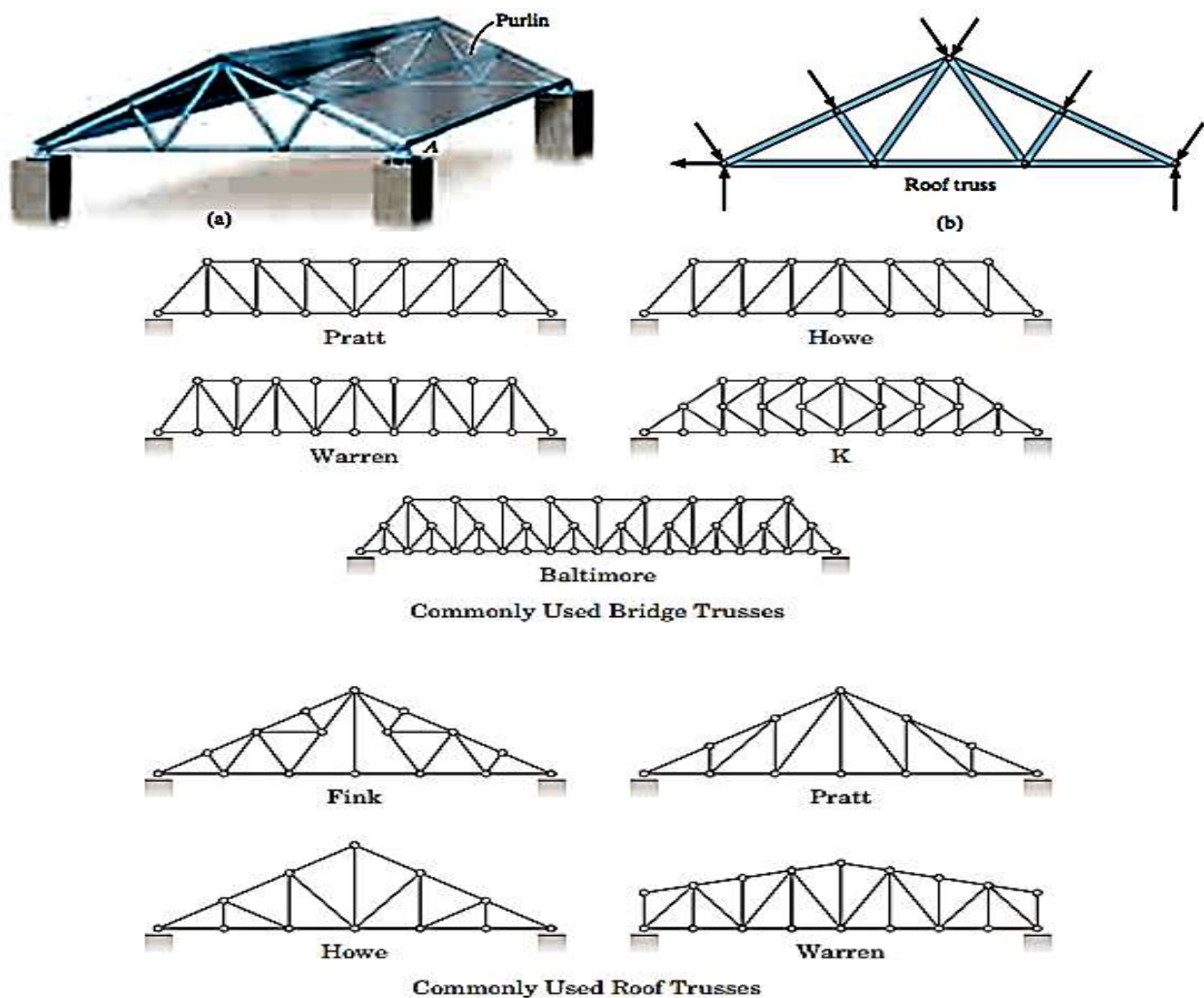
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DEBRE MARKOS INSTITUTE OF TECHNOLOGY
DEBRE MARKOS, ETHIOPIA

Chapter 4**Analysis of simple Structures****4.1. Introduction**

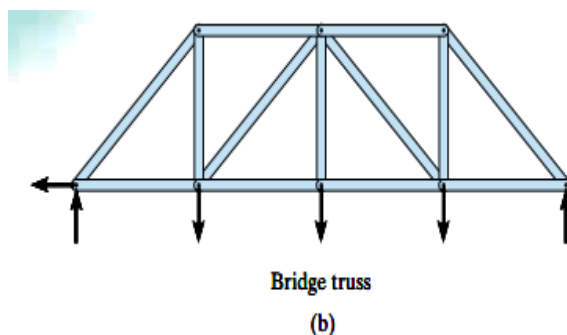
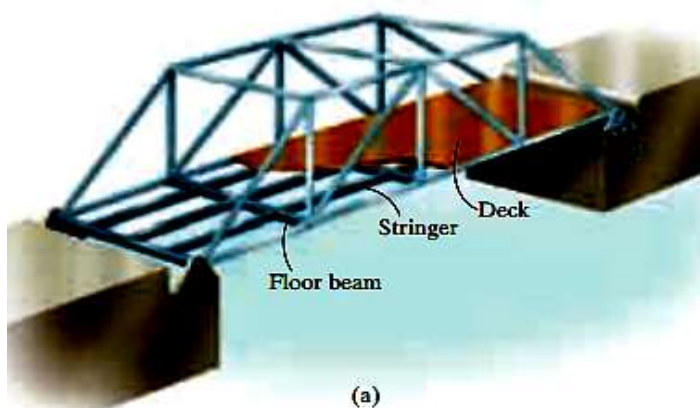
In engineering, the term structure can refer to any object that has the capacity to support and exert loads. In this chapter we consider structures composed of inter connected parts, or members. To design such a structure, or to determine whether an existing one is adequate, it is necessary to determine the forces and couples acting on the structure as a whole as well as on its individual members.

4.2. Simple Trusses

Structures that consist of straight bars pinned at the ends and are supported and loaded only at the joints where the members are connected are called trusses. It is composed of triangular elements. It is assumed that the weights of the members are negligible in comparison to the applied loads. The members commonly used in construction consist of wooden struts or metal bars. In particular, planar trusses lie in a single plane and are often used to support roofs and bridges. The truss shown in Figure a is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss at the joints by means of a series of purlins. Since this loading acts in the same plane as the truss, Figure b, the analysis of the forces developed in the truss members will be two-dimensional.



In the case of a bridge, such as shown in Figure a, the load on the deck is first transmitted to stringers, then to floor beams, and finally to the joints of the two supporting side trusses. Like the roof truss, the bridge truss loading is also coplanar, Figure b.

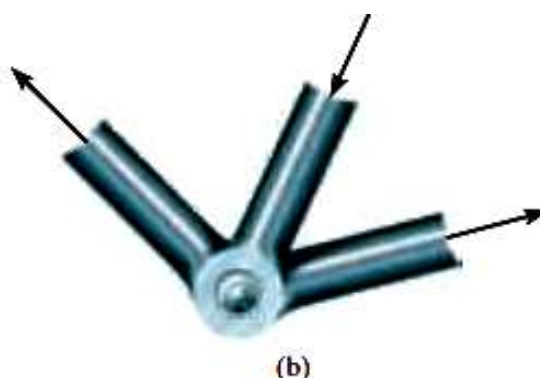
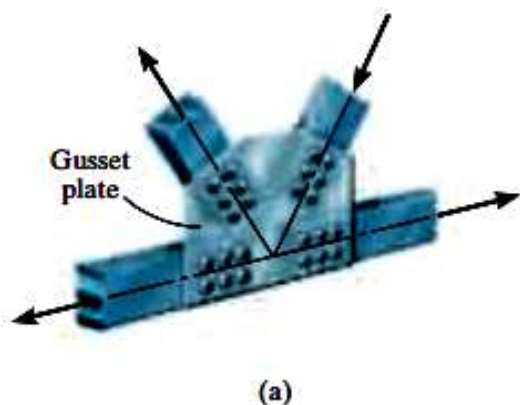


Assumptions for Design.

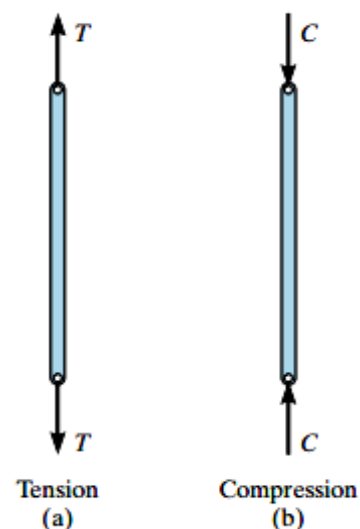
To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading.

To do this we will make two important assumptions:

- i. **All loadings are applied at the joints:** In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.
- ii. **The members are joined together by smooth pins:** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate, as shown in Figure a, or by simply passing a large bolt or pin through each of the members, Figure b. We can assume these connections act as pins provided the center lines of the joining members are concurrent, as in Figures below.



Because of these two assumptions, each truss member will act as a two-force member, and therefore the force acting at each end of the member will be directed along the axis of the member. If the force tends to elongate the member, it is a tensile force (T), Figure a; whereas if it tends to shorten the member, it is a compressive force (C), Figure b. In the actual design of a truss, it is important to state whether the nature of the force is tensile or compressive. Often, compression members must be made thicker than tension members because of the buckling or column effect that occurs when a member is in compression.



4.2.1. The method of Joint

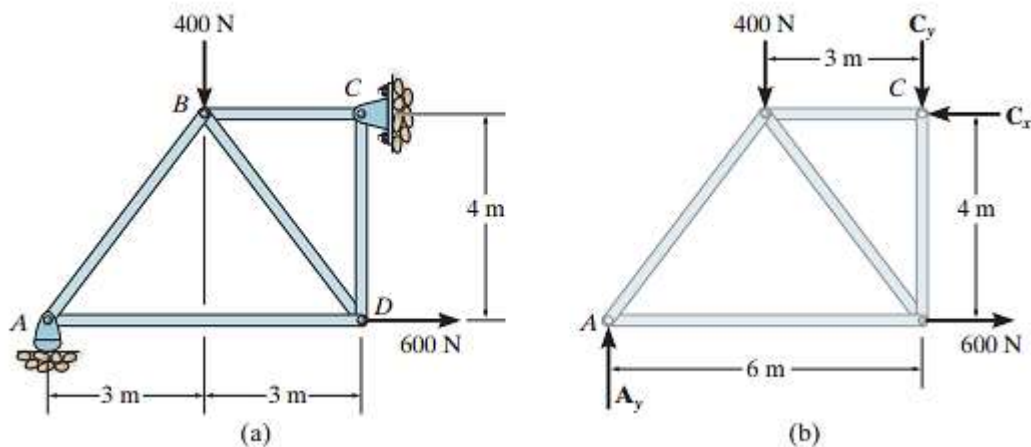
In order to analyze or design a truss, it is necessary to determine the force in each of its members. One way to do this is to use the method of joints. This method is based on the fact that if the entire truss is in equilibrium, then each of its joints is also in equilibrium. Therefore, if the free-body diagram of each joint is drawn, the force equilibrium equations can then be used to obtain the member forces acting on each joint. Since the members of a plane truss are straight two-force members lying in a single plane, each joint is subjected to a force system that is coplanar and concurrent. As a result, only $F_x = 0$ and $F_y = 0$ need to be satisfied for equilibrium. When using the method of joints, always start at a joint having at least one known force and at most two unknown forces

The following procedure provides a means for analyzing a truss using the method of joints.

- ✓ Draw the free-body diagram of a joint having at least one known force and at most two unknown forces. (If this joint is at one of the supports, then it may be necessary first to calculate the external reactions at the support.)
- ✓ Establishing the sense of an unknown force. The correct sense of direction of an unknown member force are unknown in many cases. So, the unknown member forces acting on the joint's free-body diagram are always assumed to be in tension; i.e., the forces “pull” on the pin. If this is done, then numerical solution of the equilibrium equations will yield positive scalars for members in tension and negative scalars for members in compression.
- ✓ Orient the x and y axes such that the forces on the free-body diagram can be easily resolved into their x and y components and then apply the two force equilibrium equations $F_x = 0$ and $F_y = 0$. Solve for the two unknown member forces and verify their correct sense.
- ✓ Using the calculated results, continue to analyze each of the other joints. Remember that a member in compression “pushes” on the joint and a member in tension “pulls” on the joint. Also, be sure to choose a joint having at most two unknowns and at least one known force.

Example 4.1.

Determine the force in each member of the truss shown in figure a. Indicate whether the members are in tension or compression.

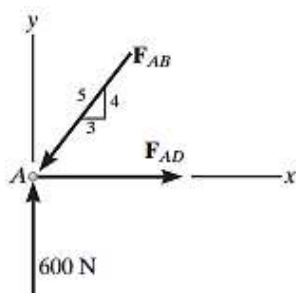

Solution

Support Reactions No joint can be analyzed until the support reactions are determined, because each joint has at least three unknown forces acting on it. A free-body diagram of the entire truss is given in figure b. Applying the equations of equilibrium, we have

$$\begin{aligned} \pm \sum F_x &= 0; & 600 \text{ N} - C_x &= 0 & C_x &= 600 \text{ N} \\ \zeta + \sum M_C &= 0; & -A_y(6 \text{ m}) + 400 \text{ N}(3 \text{ m}) + 600 \text{ N}(4 \text{ m}) &= 0 & A_y &= 600 \text{ N} \\ + \uparrow \sum F_y &= 0; & 600 \text{ N} - 400 \text{ N} - C_y &= 0 & C_y &= 200 \text{ N} \end{aligned}$$

The analysis can now start at either joint A or C. The choice is arbitrary since there are one known and two unknown member forces acting on the pin at each of these joints.

Joint A. As shown on the free-body diagram, F_{AB} is assumed to be compressive and F_{AD} is tensile.



Applying the equations of equilibrium, we have

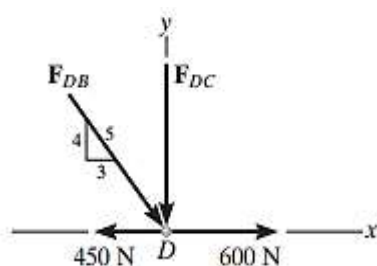
$$\begin{aligned} + \uparrow \sum F_y &= 0; & 600 \text{ N} - \frac{4}{5} F_{AB} &= 0 & F_{AB} &= 750 \text{ N (C)} \\ \pm \sum F_x &= 0; & F_{AD} - \frac{3}{5}(750 \text{ N}) &= 0 & F_{AD} &= 450 \text{ N (T)} \end{aligned}$$

Joint D. Using the result for F_{AD} and summing forces in the horizontal direction, we have

$$\pm \sum F_x = 0; \quad -450 \text{ N} + \frac{3}{5} F_{DB} + 600 \text{ N} = 0 \quad F_{DB} = -250 \text{ N}$$

The negative sign indicates that F_{DB} acts in the opposite sense to that shown in FBD. Hence,

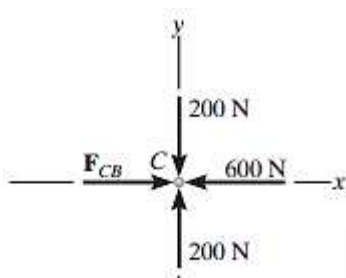
$$F_{DB} = 250 \text{ N (T)}$$



To determine F_{DC} , we can either correct the sense of F_{DB} on the free body diagram, and then apply $F_y = 0$, or apply this equation and retain the negative sign for F_{DB} , i.e.,

$$+\uparrow \Sigma F_y = 0; \quad -F_{DC} - \frac{4}{5}(-250 \text{ N}) = 0 \quad F_{DC} = 200 \text{ N} \quad (\text{C})$$

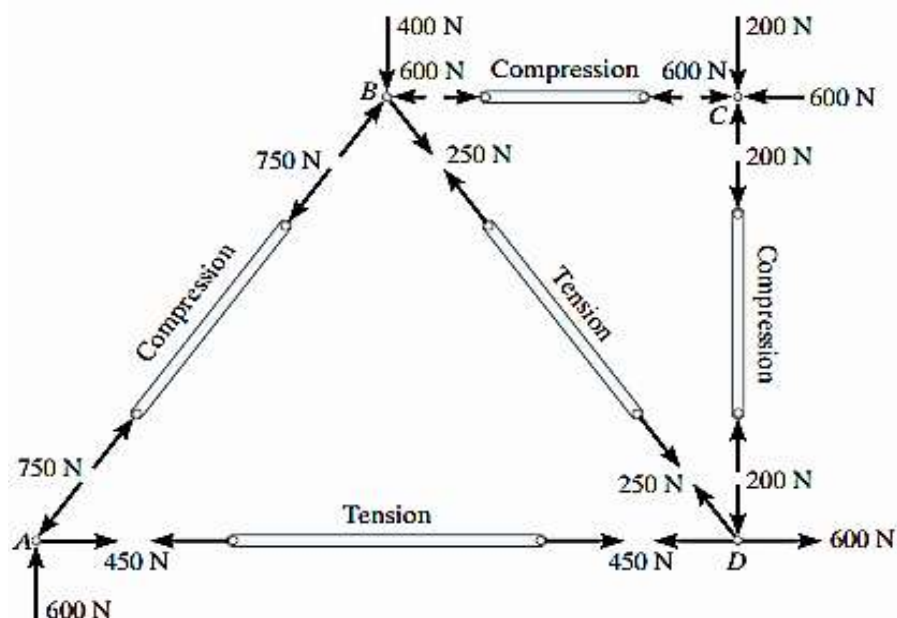
Joint C.



$$+\rightarrow \Sigma F_x = 0; \quad F_{CB} - 600 \text{ N} = 0 \quad F_{CB} = 600 \text{ N} \quad (\text{C})$$

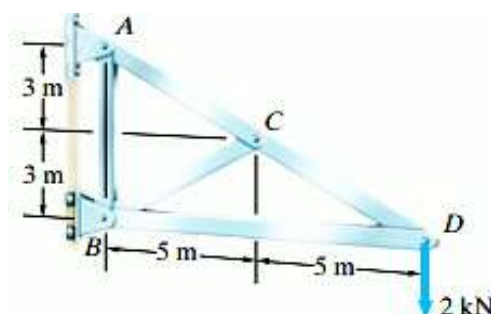
$$+\uparrow \Sigma F_y = 0; \quad 200 \text{ N} - 200 \text{ N} \equiv 0 \quad (\text{check})$$

NOTE: The analysis is summarized in figure, which shows the free body diagram for each joint and member.



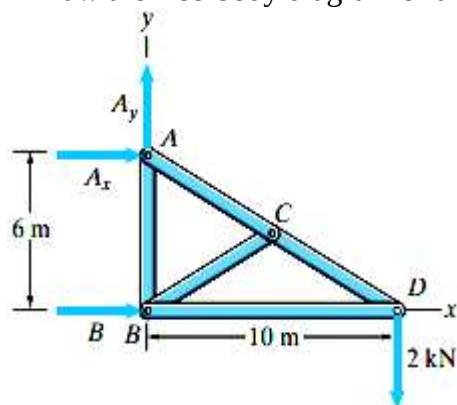
Example 4.2.

Determine the axial forces in members AB and AC of the truss.



Solution

- ✓ Draw the free-body diagram of the entire truss and apply the equilibrium equations



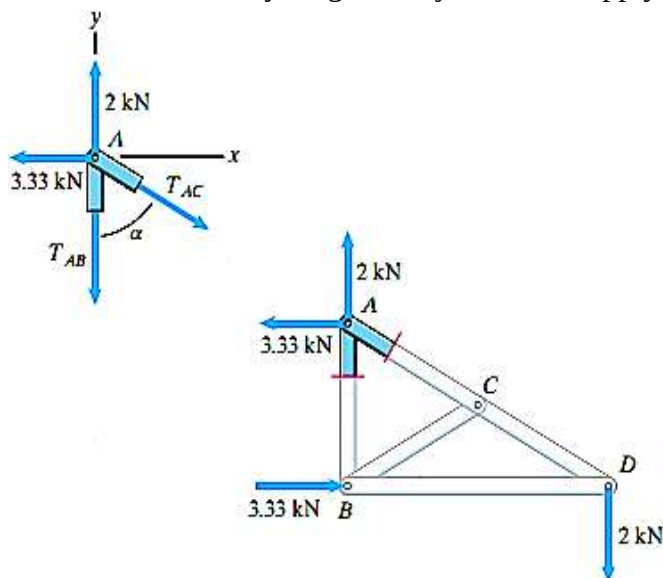
$$\Sigma F_x = A_x + B = 0,$$

$$\Sigma F_y = A_y - 2 \text{ kN} = 0,$$

$$\Sigma M_{\text{point } B} = -(6 \text{ m})A_x - (10 \text{ m})(2 \text{ kN}) = 0.$$

Solving yields $A_x = -3.33 \text{ kN}$, $A_y = 2 \text{ kN}$,
and $B = 3.33 \text{ kN}$.

- ✓ Draw the free-body diagram of joint A and apply the equilibrium equations



The angle $\alpha = \arctan(5/3) = 59.0^\circ$.

$$\sum F_x = T_{AC} \sin \alpha - 3.33 \text{ kN} = 0,$$

$$\sum F_y = 2 \text{ kN} - T_{AB} - T_{AC} \cos \alpha = 0.$$

Solving yields $T_{AB} = 0$ and $T_{AC} = 3.89 \text{ kN}$.

The axial force in member AB is zero and the axial force in member AC is 3.89 kN in tension, which we write as

AB: zero, AC: 3.89 kN (T)

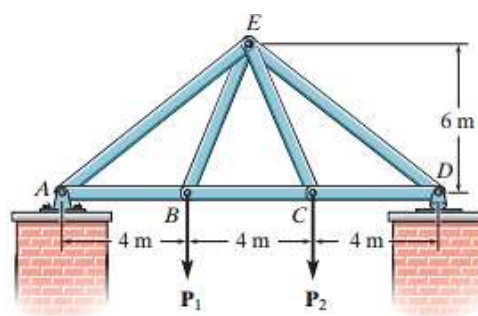
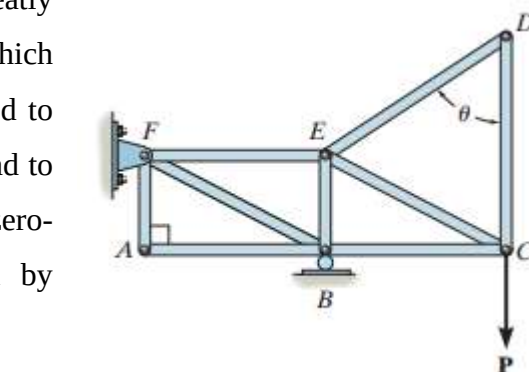
Zero Force Member

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support no loading. These zero-force members are used to increase the stability of the truss during construction and to provide added support if the loading is changed. The zero-force members of a truss can generally be found by inspection of each of the joints.

Exercise 4.1.

Determine the force in each member of the truss and state if the members are in tension or compression.

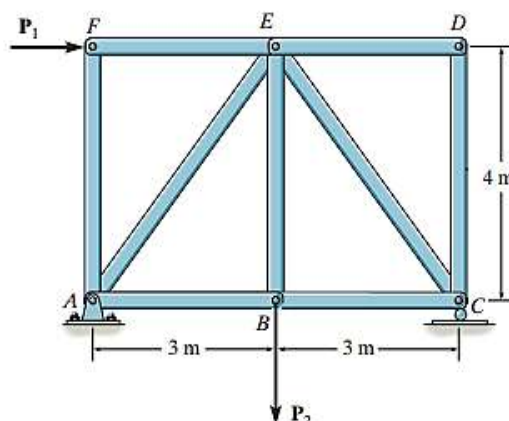
Set $P_1 = 6 \text{ kN}$, $P_2 = 9 \text{ kN}$.



Exercise 4.2.

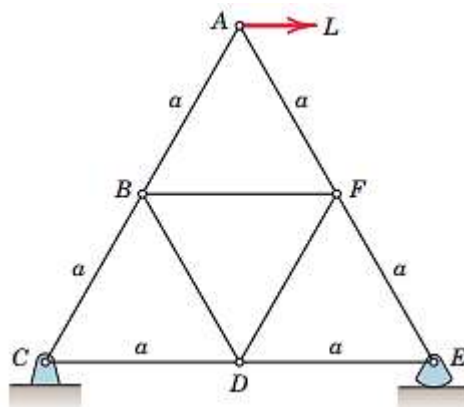
Determine the force in each member of the truss and state if the members are in tension or compression. Set

$P_1 = 30 \text{ kN}$, $P_2 = 15 \text{ kN}$.



Exercise 4.3.

The equiangular truss is loaded and supported as shown. Determine the forces in all members in terms of the horizontal load L

**4.2.2. Method of Section**

This method is based on the principle that if the truss is in equilibrium then any segment of the truss is also in equilibrium. It can be used to “cut” or section the members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, we can then apply the equations of equilibrium to that part to determine the member forces at the “cut section.” Since only three independent equilibrium equations ($F_x = 0$, $F_y = 0$, $M_O = 0$) can be applied to the free-body diagram of any segment, then we should try to select a section that, in general, passes through not more than three members in which the forces are unknown.

The forces in the members of a truss may be determined by the method of sections using the following procedure.

- **Free-Body Diagram.**

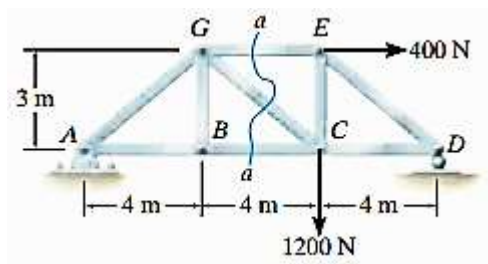
- ✓ Make a decision on how to “cut” or section the truss through the members where forces are to be determined.
- ✓ Before isolating the appropriate section, it may first be necessary to determine the truss’s support reactions. If this is done then the three equilibrium equations will be available to solve for member forces at the section.
- ✓ Draw the free-body diagram of that segment of the sectioned truss which has the least number of forces acting on it.
- ✓ Use one of the two methods described above for establishing the sense of the unknown member forces.

- **Equations of Equilibrium.**

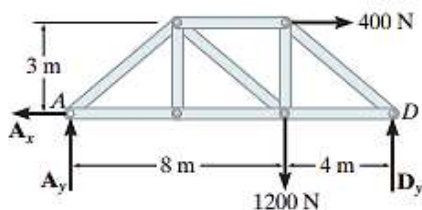
- ✓ Moments should be summed about a point that lies at the intersection of the lines of action of two unknown forces, so that the third unknown force can be determined directly from the moment equation.
- ✓ If two of the unknown forces are parallel, forces may be summed perpendicular to the direction of these unknowns to determine directly the third unknown force.

Example 4.3.

Determine the force in members GE, GC, and BC of the truss shown in figure. Indicate whether the members are in tension or compression.

**Solution**

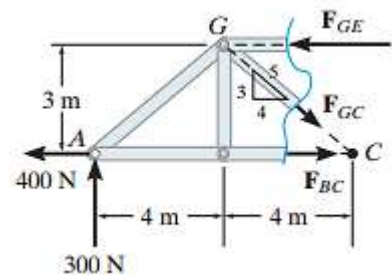
Section a-a in figure above has been chosen since it cuts through the three members whose forces are to be determined. In order to use the method of sections, however, it is first necessary to determine the external reactions at A or D. Why? A free-body diagram of the entire truss is shown in figure. Applying the equations of equilibrium, we have



$$\begin{aligned} \rightarrow \Sigma F_x &= 0; & 400 \text{ N} - A_x &= 0 & A_x &= 400 \text{ N} \\ \zeta + \Sigma M_A &= 0; & -1200 \text{ N}(8 \text{ m}) - 400 \text{ N}(3 \text{ m}) + D_y(12 \text{ m}) &= 0 & D_y &= 900 \text{ N} \\ +\uparrow \Sigma F_y &= 0; & A_y - 1200 \text{ N} + 900 \text{ N} &= 0 & A_y &= 300 \text{ N} \end{aligned}$$

Free-Body Diagram. For the analysis the free-body diagram of the left portion of the sectioned truss will be used, since it involves the least number of forces, figure.

Equations of Equilibrium. Summing moments about point G eliminates F_{GE} and F_{GC} and yields a direct solution for F_{BC} .



$$\begin{aligned} \zeta + \Sigma M_G &= 0; & -300 \text{ N}(4 \text{ m}) - 400 \text{ N}(3 \text{ m}) + F_{BC}(3 \text{ m}) &= 0 \\ F_{BC} &= 800 \text{ N (T)} \end{aligned}$$

In the same manner, by summing moments about point C we obtain a direct solution for F_{GE} .

$$\begin{aligned} \zeta + \Sigma M_C &= 0; & -300 \text{ N}(8 \text{ m}) + F_{GE}(3 \text{ m}) &= 0 \\ F_{GE} &= 800 \text{ N (C)} \end{aligned}$$

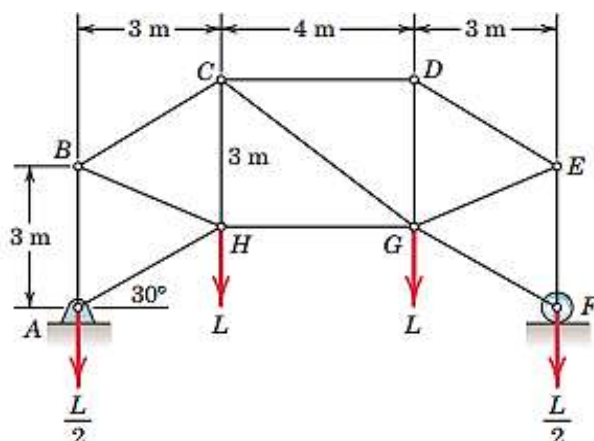
Since F_{BC} and F_{GE} have no vertical components, summing forces in the y direction directly yields F_{GC} , i.e.,

$$\begin{aligned} +\uparrow \Sigma F_y &= 0; & 300 \text{ N} - \frac{3}{5}F_{GC} &= 0 \\ F_{GC} &= 500 \text{ N (T)} \end{aligned}$$

NOTE: Here it is possible to tell, by inspection, the proper direction for each unknown member force. For example, $\Sigma M_C = 0$ requires F_{GE} to be compressive because it must balance the moment of the 300-N force about C.

Exercise 4.4.

Determine the forces in members CG and GH of the symmetrically loaded truss.

**Exercise 4.5.**

What is meant by **space truss**? Discuss briefly and Illustrate it using figures.

4.3. Frames and Machines

Frames and machines are two types of structures which are often composed of pin-connected multi-force members, i.e., members that are subjected to more than two forces.

A frame is a structure that contains one or more three-force and/or multi-force members. Typically, a frame is fully fixed in space and uses a stationary arrangement of members with the goal of supporting forces that are applied to it.

A machine is an arrangement of members where typically the members can have significant motion relative to one another. The usual goal of a machine is transmission of motion and/or force. While a machine can be composed entirely of two-force members, in which case it may be analyzed as a truss, in this section we are interested in machines that contain one or more multi-force members.

The joint reactions on frames or machines (structures) composed of multi-force members can be determined using the following procedure.

Free-Body Diagram.

- ✓ Draw the free-body diagram of the entire frame or machine, a portion of it, or each of its members. The choice should be made so that it leads to the most direct solution of the problem.
- ✓ Identify the two-force members. Remember that regardless of their shape, they have equal but opposite collinear forces acting at their ends.
- ✓ When the free-body diagram of a group of members of a frame or machine is drawn, the forces between the connected parts of this group are internal forces and are not shown on the free-body diagram of the group.

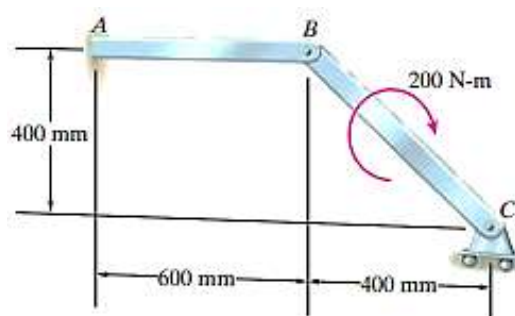
- ✓ Forces common to two members which are in contact act with equal magnitude but opposite sense on the respective free-body diagrams of the members.
- ✓ In many cases it is possible to tell by inspection the proper sense of the unknown forces acting on a member; however, if this seems difficult, the sense can be assumed.
- ✓ Remember that once the free-body diagram is drawn, a couple moment is a free vector and can act at any point on the diagram. Also, a force is a sliding vector and can act at any point along its line of action.

Equations of Equilibrium.

- ✓ Count the number of unknowns and compare it to the total number of equilibrium equations that are available. In two dimensions, there are three equilibrium equations that can be written for each member.
- ✓ Sum moments about a point that lies at the intersection of the lines of action of as many of the unknown forces as possible.
- ✓ If the solution of a force or couple moment magnitude is found to be negative, it means the sense of the force is the reverse of that shown on the free-body diagram.

Example 4.4.

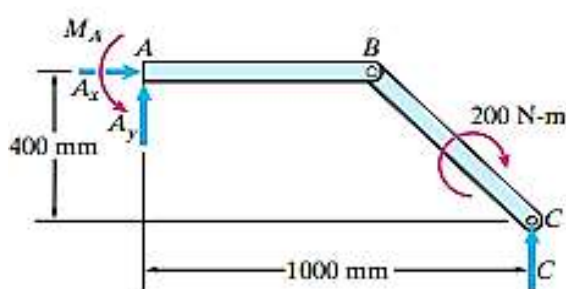
Determine the forces and couples acting on the members of the frame



Solution

We will first draw a free-body diagram of the entire frame, treating it as a single object, and attempt to determine the reactions at the supports. We will then draw free-body diagrams of the individual members and apply the equilibrium equations to determine the forces and couples acting on them.

Draw the free-body diagram of the entire frame and apply the equilibrium equations



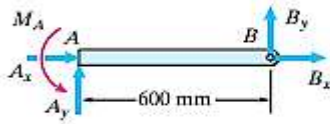
$$\sum F_x = A_x = 0,$$

$$\sum F_y = A_y + C = 0,$$

$$\sum M_{\text{point A}} = M_A - 200 \text{ N}\cdot\text{m} + (1.0 \text{ m})C = 0.$$

The reaction $A_x=0$, but A_y , C , and M_A cannot be determined from these equations. The free-body diagram of the entire frame is statically indeterminate

Draw the free-body diagrams of the individual members



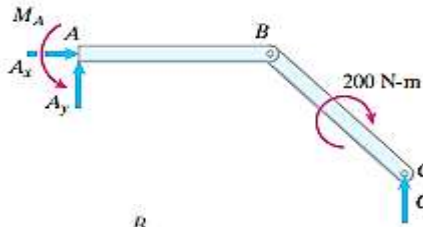
Apply equilibrium to member B

$$\sum F_x = -B_x = 0,$$

$$\sum F_y = -B_y + C = 0,$$

$$\sum M_{\text{point } B} = -200 \text{ N}\cdot\text{m} + (0.4 \text{ m})C = 0.$$

Solving yields $B_x=0$, $B_y=500 \text{ N}$, and $C=500 \text{ N}$

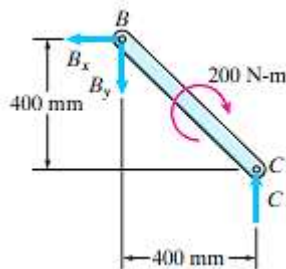


Apply equilibrium to member AB

$$\sum F_x = A_x + B_x = 0,$$

$$\sum F_y = A_y + B_y = 0,$$

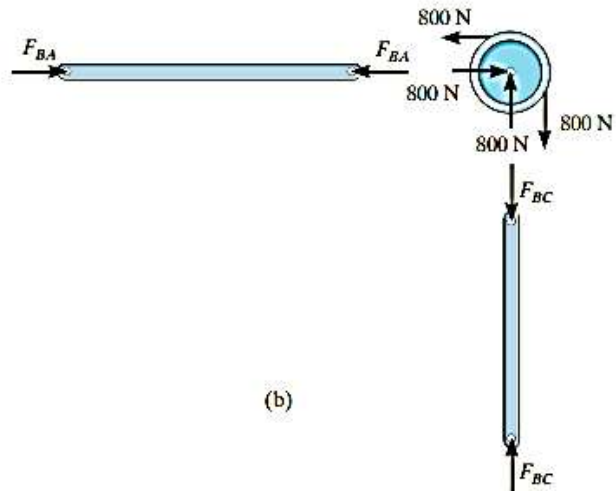
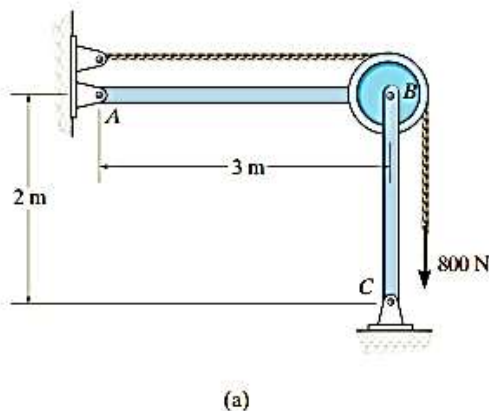
$$\sum M_{\text{point } A} = M_A + (0.6 \text{ m})B_y = 0.$$



Because A_x , B_x , and B_y have already been determined, these equations can be solved for A_y and M_A . The results are $A_y 500 \text{ N}$ and $M_A=300 \text{ N}\cdot\text{m}$, which completes the solution

Example 4.5.

Determine the force the pins at A and B exert on the two-member frame shown in figure a.



Solution

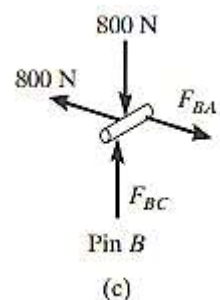
Free-Body Diagrams.

By inspection AB and BC are two-force members. Their free-body diagrams, along with that of the pulley, are shown in figure b. In order to solve this problem, we must also include the free-body diagram of the pin at B because this pin connects all three members together, figure c.

Equations of Equilibrium: Apply the equations of force equilibrium to pin B.

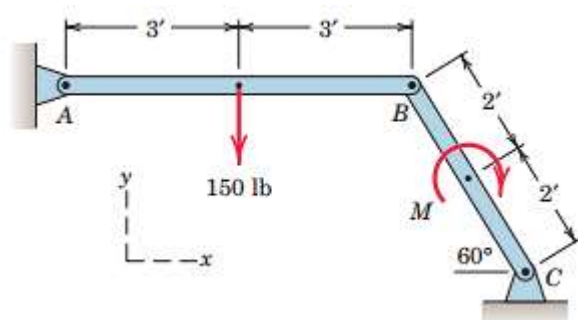
$$+\rightarrow \sum F_x = 0; \quad F_{BA} - 800 \text{ N} = 0; \quad F_{BA} = 800 \text{ N}$$

$$+\uparrow \sum F_y = 0; \quad F_{BC} - 800 \text{ N} = 0; \quad F_{BC} = 800 \text{ N}$$

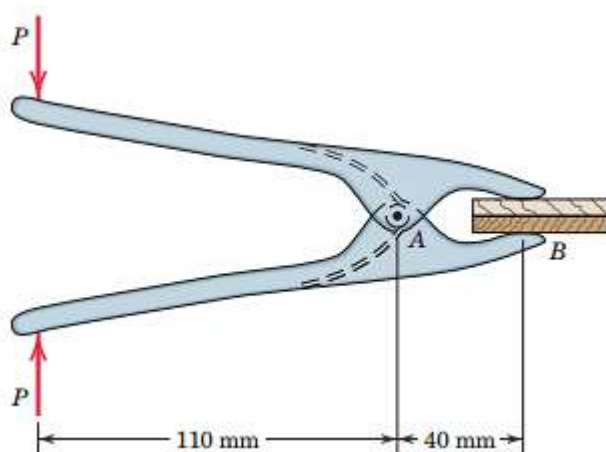


Exercise 4.6.

For what value M of the clockwise couple will the horizontal component A_x of the pin reaction at A be zero? If a couple of that same magnitude M were applied in a counterclockwise direction, what would be the value of A_x ?

**Exercise 4.7.**

In the spring clamp shown, an internal spring is coiled around the pin at A and the spring ends bear against the inner surfaces of the handle halves in order to provide the desired clamping force. In the position shown, a force of magnitude P is required to release the clamp. Determine the compressive force at B if $P=0$.

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